Social Networks 63 (2020) 47–55

Contents lists available at ScienceDirect

Social Networks

journal homepage: www.elsevier.com/locate/socnet

Universal evolution patterns of degree assortativity in social networks

Bin Zhou[a], Xin Lu[b,c,*], Petter Holme[d]

[a] School of Economics and Management, Jiangsu University of Science and Technology, Zhenjiang, 212003, China
[b] School of Business, Central South University, Changsha, 410083, China
[c] College of Systems Engineering, National University of Defense Technology, Changsha, 410073, China
[d] Tokyo Tech World Research Hub Initiative (WRHI), Institute of Innovative Research, Tokyo Institute of Technology, Nagatsuta-cho 4259, Midori-ku, Yokohama, Kanagawa, 226-8503, Japan

ARTICLE INFO

Keywords:
Degree assortativity
Evolution pattern
Social networks
Pareto wealth distribution
Bidirectional preferential attachment

ABSTRACT

Degree assortativity characterizes the propensity for large-degree nodes to connect to other large-degree nodes and low-degree to low-degree. It is important to describe the forces forming the network and to predict the behavior of dynamic systems on the network. To understand the evolutionary dynamics of degree assortativity, we collect a variety of empirical temporal social networks, and find that there is a universal pattern that the degree assortativity increases at the beginning of evolution and then decreases to a long-lasting stable level. We develop a bidirectional selection model to re-construct the evolution dynamic. In our model, we assume each individual has a social status that—in analogy to Pareto's wealth distribution —follows a power-law distribution. We assume the social status determines the probability of an interaction between two actors. By varying the ratio of link establishment from within the same status level to across different status levels, the simulated network can be tuned to be assortative or disassortative. This suggests that the rise-and-fall pattern of degree assortativity is a consequence of the different network-forming forces active at different mixing of status. Our simulations indicate that Pareto social status distribution in the population may drive the social evolution in a way of self-optimization to promote the social interaction among individuals and the status gap plays an important role for the assortativity of the social network.

Introduction

The rapid development of social networking platforms, such as Facebook, Twitter, WeChat, Sina microblogs, etc., has brought together millions of users to share their interests and maintain social interactions (Faraj et al., 2011). Understanding the structure of social networks is important for comprehending the evolutionary, functional, and dynamical processes taking place in these systems (Barabási and Albert, 1999; Strogatz, 2001; Barrat et al., 2008). Thus, the statistics and dynamics of social networking services have attracted a lot of attention in the past years. (Ohtsuki et al., 2006; Opsahl et al., 2010; Del Vicario et al., 2017; Becker et al., 2017; Kim and Hastak, 2018). Degree assortativity is one of the most important structural measurement for the study of social networks, as it has been revealed that dynamics of networked systems, including such as synchronization, percolation, social organization, network robustness (Newman and Park, 2003; Boguná et al., 2003; Holme and Zhao, 2007; Di Bernardo et al., 2007; Zhou et al., 2012) are all affected by degree assortativity. Degree assortativity quantifies the tendency of nodes of similar degree to be connected by a link, and it is traditionally defined via Pearson's correlation coefficient

$$r = \frac{\langle ij\rangle - \langle i\rangle\langle j\rangle}{\sqrt{\langle i^2\rangle - \langle i\rangle^2}\sqrt{\langle j^2\rangle - \langle j\rangle^2}} \tag{1}$$

where i and j are the degrees at the two ends of a link and the $\langle\bullet\rangle$ notation represents the average over all links. The assortativity r lies in the range $-1 \le r \le 1$. When $r > 0$, the network has an assortative mixing pattern, and when $r < 0$, it shows disassortative mixing. An uncorrelated network exhibits the neutral degree-mixing pattern with $r = 0$ (Newman, 2002). One leading hypothesis is that most social networks have an assortative mixing (Newman and Park, 2003), but disassortative networks have also been observed, in particular those derived from online interaction (Holme et al., 2004; Noldus and van Mieghem, 2015). The definition in Eq. (1) is not without problems. In networks with heavy tailed degree distributions, it can show spurious correlations (Litvak and van der Hofstad, 2013). It can also suffer from large finite-size effects so that a process that provably will lead to an assortative network shows disassortativity for even rather large

* Corresponding author.
E-mail address: xin.lu@flowminder.org (X. Lu).

https://doi.org/10.1016/j.socnet.2020.04.004

networks (Xulvi-Brunet and Sokolov, 2005). For this paper, we will just take Eq. (1) as our definition of assortativity and leave it for a future study how more elaborate approaches would change the picture.

Assortativity plays an important role in information diffusion (Jiang et al., 2016) and disease spreading (Eguiluz and Klemm, 2002; Badham and Stocker, 2010). On networks with community structure, the assortativity within a community can differ much from the assortativity of the whole network. This fact can be used to improve community detection methods (Ciglan et al., 2013). The geographical distances between countries in the world put strong constraint on the international trade network. It has a peculiar structure—disassortative at long distances while it being assortative nearby (Abbate et al., 2017). Under a general class of geometric network growth mechanisms by homogeneous attachment, the assortativity of the network can be tuned between assortativity and disassortativity depending on the degree order in which nodes appear in the network (Murphy et al., 2018). In conclusion, the assortativity can be very helpful to understand the structure and function of social networks and can also help solving management problems in social networks. Therefore, the research on the time evolution of the degree assortativity is important.

Several models have been proposed to simulate the assortativity of social networks (Catanzaro et al., 2004; Toivonen et al., 2006; Li et al., 2014) and find that individuals' personal motives and the influence within neighborhood are the key mechanisms for the assortativity in social networks. A mutual attraction model which can generate weighted simulation network was proposed to mimic assortative and disassortative social networks (Wang et al. 2006). Statistically speaking, the assortativity of a network depends on the balance between three structural factors: transitivity (clustering), intermodular connectivity, and relative branching. The first two factors perform a positive contribution to the assortativity of a network, while branching is more likely associated with disassortative networks (Estrada, 2011). The concept of the two-walks degree assortativity was proposed to account for the effect of second neighbors to a given node in a network. The new index includes more structural information of the networks than the old one and a class of networks which are degree disassortative and two-walks degree assortative are observed (Allen-Perkins et al., 2017).

Most of the research on assortativity have only showed the static topology characteristics of social networks at a certain moment. The social networks are dynamic and evolving from the initial state to the final equilibrium as time goes on, so it is vital to study the evolutionary dynamics of the assortativity to obtain a deeper understanding of the evolution pattern of the assortativity in social networks. The study of social networks in *Wealink* and *Pussokram* reveals that the assortativity arises quickly at the beginning of network formation and then decreases to a stable state after a long-term evolution (Hu and Wang, 2009; Holme et al., 2004). In our empirical demonstration, the same evolution pattern of degree assortativity is also found in social networks in *Wikipedia*, *Renren*, *Primary School*, *High School*, *Wikipedia English*, *Wikipedia Italian* and *Wikipedia German*. The universal evolution pattern that the degree assortativity increases at the beginning of evolution and then decreases to a long-term stable level intrigue us with the following questions: Why is there such a universal pattern? Is there a generic explanation for this? As far as we know, there are neither a generic mechanism nor a specific model to explain and reproduce the universal evolution pattern of degree assortativity. Our work seeks to shed some light on the question. We assume that the individual social status level in a real society plays an important role in the evolution process of social networks. We furthermore assume that social interactions are easier to be established among individuals of higher social status—an effect that we call the bidirectional preferential attachment. In the evolutionary process of social networks, when the interactions among individuals with the same status level are dominant, the network tends to be assortative, and when the interactions among individuals with different status level are dominant, the network tends to be disassortative. Based on the individual status information and the bidirectional preferential attachment, we propose a model with a control parameter to reproduce the universal evolution pattern of degree assortativity in social networks.

This rest of the paper is organized as follows: First, we make the empirical demonstration of nine social networks and present the universal evolution pattern of the assortativity on these networks. Second, we analyze the inherent features of human social interaction in social networks and propose the mechanisms to explain the sinusoidal evolution characteristic of the assortativity. Third, we provide a model based on the blend mechanisms and the analytical solutions of both degree distribution and assortativity of the model are presented. Fourth, the simulations based on the empirical data are done to reproduce the universal evolution pattern of the assortativity. Fifth, we test the sensitivity of the model with comprehensive simulations under various parameter settings. Finally, we discuss the significance of the work and conclude with a summary of our results.

Empirical demonstration and analysis

Data description

Each social network is composed of the set of nodes, which represent individual users, and the set of links, which represent a type of social interaction or friend relationship between the nodes. For the purpose of this study, we assume that all links are undirected. To study the evolutionary pattern of online social networks, we collect nine different datasets, all covering the time span from the initial creation stage to the later relevant stable stage. These datasets are from *Wealink* (www.wealink.com), *Wikipedia* (www.wikipedia.org), *Renren* (www.renren.com), *Pussokram* (www.pussokram.com), *Primary School*, *High School* (www.sociopatterns.org), *Wikipedia English, Wikipedia Italian* and *Wikipedia German* (http://konect.uni-koblenz.de).

Wealink is a Chinese version of LinkedIn, which provides professional social networking services and act as a platform for people posting and looking for job position. The data set begun on 11 May 2005 and covers a duration of 323 days. The network includes 11,262 nodes and 14,050 links.

Wikipedia is the largest collaborative encyclopedic system in the world and it precisely records all online collaborative activities among users. The data set begun on 20 Feb 2001 and covers a duration of 353 days. The network includes 16,435 nodes and 29,032 links.

Renren is one of the largest social networks in China and it provides a platform to make people establish social interaction with each other. The data set begun on 21 Nov 2005 and covers a duration of 98 days. The network includes 5554 nodes and 20,121 links.

Pussokram was a Swedish Internet community primarily intended for romantic communication and targeted at adolescents and young adults. The data set begun on February 13, 2001 and covers a duration of 124 days. The network includes 14,547 nodes and 49,931 links.

Primary School and *High School* is two evolutionary networks of social interaction. The data is from the data platform. The data of *Primary School* includes 238 nodes and 5541 links with a duration of 515 min (Stehlé et al., 2011). The data of High School includes 295 nodes and 2162 links with a duration of 539 min (Mastrandrea et al., 2015).

Wikipedia English, *Wikipedia Italian* and *Wikipedia German* are networks including the social interaction of hyperlinks between user articles of the English Wikipedia, the Italian Wikipedia and the German Wikipedia, respectively. The data of *Wikipedia English* includes 100,312 nodes and 693,796 links with a duration of 115 months; the data of *Wikipedia Italian* includes 1,204,009 nodes and 15,710,767 links with a duration of 102 months; and the data of *WikipediaGerman*includes 1,655,808 nodes and 14,345,088 links with a duration of 96 months.

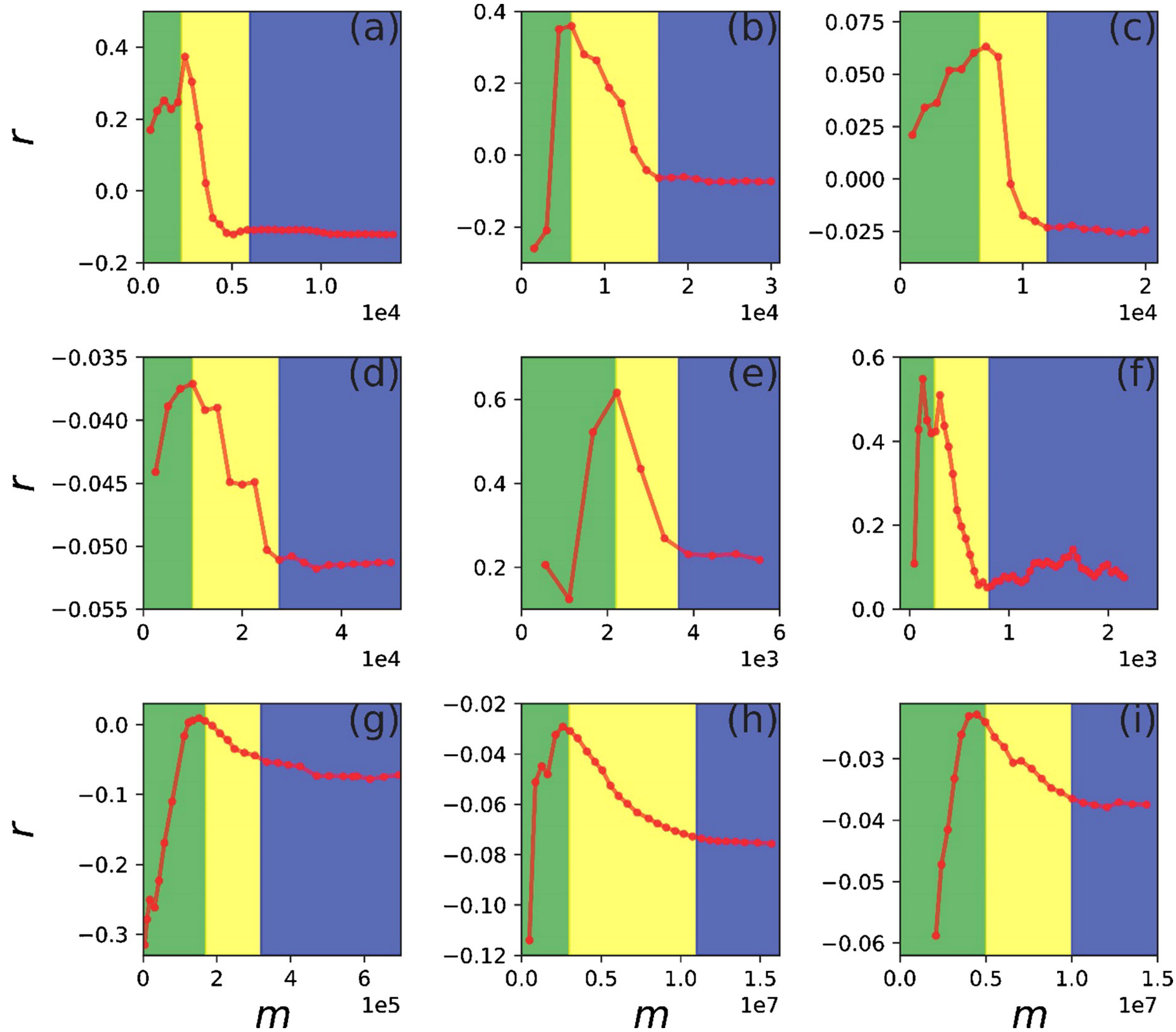

Fig. 1. The evolutions of the assortativity in the nine social networks. The subfigures (a)-(i) shows the evolution process of the assortativity in *Wealink*, *Wikipedia*, *Renren*, *Pussokram*, *Primary School*, *High School*, *Wikipedia English*, *Wikipedia Italian* and *Wikipedia German* respectively. Green: increasing area; Yellow: decreasing area; Blue: stable area.

Evolution of degree assortativity

Fig. 1 show the evolution of assortativity in *Wealink*, *Wikipedia*, *Renren*, *Pussokram*, *Primary School*, *High School*, *Wikipedia English*, *Wikipedia Italian* and *Wikipedia German* respectively. In sub-figs (a)–(i), we can see that in all these nine social networks, the evolution of the assortativity follow the same pattern, which is formed of three phases.

Phase one (the increase): the assortativity rises in the initial evolution stage of the network creation with a limited number of edges; Phase two (the decrease): the assortativity begins to drop down when the number of links exceeds a certain level; Phase three (the plateau): the assortativity reaches a stable value and long-lasts with the increase of links in the network. The rise and down of assortativity in these networks reflect the rule for establishing social interactions among individuals. In the initial evolutionary stages of the nine social networks, the number of the registered individuals rapidly increases, and complex social interactions are established among individuals. Although the nine social networks are greatly different in network size and evolution time, they all show striking similarity in the evolution pattern of the assortativity. What basic rules of establishing social interactions result in the universal evolution characteristic of assortativity in the nine social networks? Based on the inherent features of the social networks, we will analyze the evolutionary mechanisms of the social networks and explain the universal evolution pattern of the assortativity.

Analysis of the evolutionary mechanisms

We start by considering a few potential mechanisms for the evolution of social networks. First, a common feature of social network modeling is to keep the size (number of individuals) as an independent parameter. Second, the status of individuals should play an important role in the formative processes of social networks. Empirical research has shown that individual wealth is distributed according to Pareto distribution (Arnold, 2015; Newman, 2005; Levy and Solomon, 1997; Klass et al., 2006). The Pareto distribution is a power law probability distribution that is used in description of social, scientific, geophysical, actuarial, and many other types of observable phenomena (Pareto, 1964; Arnold, 2015). In this paper, we make the assumption that this extends to general features—we call these features social status (but caution the reader that this term that is used in many ways in the literature (Bordieu, 1984))—determining the popularity of a person (Lee and Holme, 2017). Third, when an individual makes a request to establish social interaction, the inquired individual can choose to accept or reject the request. There might be a tendency to accept invites from actors of higher status leading to a well-connected rich club (Colizza et al., 2006; McAuley et al., 2007; Opsahl et al., 2008; Vaquero and

Cebrian, 2013). The process of establishing social interaction among individuals could thus be described as a bidirectional preferential selection process, or bidirectional preferential attachment mechanism. More importantly, in the evolutionary process of social networks, the social interactions should happen simultaneously among individuals with the same status level and among individuals with different status level. When the social interactions among individuals with the same status level dominate, the network trends to be assortative, whereas when the social interactions among individuals with different status level dominate, the network tends to be disassortative.

Model and analytical solution

Deriving the bidirectional selection model

To model the evolutionary mechanism of assortativity in social networks, based on the previous research (Zhou et al., 2018), we propose a bidirectional selection model with individual status information. We consider a network with a fixed number of individuals N. To model individual status information, each individual is assigned a status ω according to the power-law distribution $p(\omega) = c\omega^{-\alpha}$, in which c is the normalized constant, $\omega \in [a, b]$ is a positive integral with $a \le b$, and α is the power exponent. The average status $\bar{\omega}$ of an individual is $\bar{\omega} = \sum_{\omega=a}^{b} \omega c\omega^{-\alpha}$. The rules of establishing social interactions among individuals are as follows.

1 To model the bidirectional preferential attachment mechanism, at each time step, we independently chose two individuals. The probability that an individual i will be chosen depends on the status ω_i of the individual, so that $\Pi(\omega_i) = \omega_i/N\bar{\omega}$ and the probability that two individuals i and j are chosen in one timestep is $\Pi(\omega_i, \omega_j) = 2\omega_i\omega_j/(N\bar{\omega})^2$.

2 To model the assortativity or disassortativity of social network, we assume the probability that two individuals i and j are chosen and establish social interaction at each time step is $\Lambda(\omega_i, \omega_j) = \Pi(\omega_i, \omega_j)\Delta(\omega_i, \omega_j)$, in which the control component Δ follows that

$$\Delta = \rho e^{(1-\lambda_1)\beta} + (1-\rho)e^{(a/b-\lambda_2)\gamma} \tag{2}$$

where $\rho \in [0,1]$, $\beta \ge 0$ and $\gamma \ge 0$ are model parameters, and $\lambda_1 = \max(\omega_i/\omega_j, \omega_j/\omega_i)$, $\lambda_2 = \min(\omega_i/\omega_j, \omega_j/\omega_i)$, so that $\lambda_1 \in [1, b/a]$, $\lambda_2 \in [a/b, 1]$.

After T time steps the simulation is done. We can see that the model presented in Zhou et al. (2018) is a special case of the present model when $\beta = 0$, $\gamma = 0$ and $\Delta = 1$. While the current model is extended to be able to adjust the assortativity of the simulation network with the control component.

Basic observations of the model behavior

In this section, we go over some observations about the model behavior that follows immediately from the definition. When $\omega_i = \omega_j$, $\lambda_1 = 1$ and $e^{(1-\lambda_1)\beta} = 1$. The smaller the status gap between ω_i and ω_j is, the smaller λ_1 is, but the larger $e^{(1-\lambda_1)\beta}$ is. This means that individuals with similar status level are more likely to establish social interaction, which drives the network to be assortative. So, $e^{(1-\lambda_1)\beta}$ controls the assortativity of the simulation social network in the evolution process, and $e^{(1-\lambda_1)\beta} \in [0,1]$. The larger β is, the smaller $e^{(1-\lambda_1)\beta}$ is, then the two chosen individuals are less likely to establish an assortativity connection by social status, i.e., β is negatively related to the status assortativity of the simulation network. Conversely, when $\omega_i = a$ and $\omega_j = b$, $\lambda_2 = a/b$ and $e^{(a/b-\lambda_2)\gamma} = 1$. The larger the status gap between ω_i and ω_j is, the smaller λ_2 is, but the larger $e^{(a/b-\lambda_2)\gamma}$ is. This means that individuals with different status level are more likely to establish social interaction, which drives the network to be disassortative. So, $e^{(a/b-\lambda_2)\gamma}$ controls the disassortativity of the simulation social network in the evolution process, and $e^{(a/b-\lambda_2)\gamma} \in [0,1]$. The bigger γ is, the smaller is $e^{(a/b-\lambda_2)\gamma}$. Thus two chosen individuals are less likely to establish a disassortativity connection by social status, i.e., γ is negatively related to the status disassortativity of the network. The last important parameter ρ represents that two chosen individuals will make contribution to the assortativity of the simulation social network with probability ρ, and with probability $1-\rho$ the two chosen individuals will make contribution to the disassortativity of the simulation social network. The parameters ρ, β and γ can be constants or functions of evolutionary time t. Especially, if $\beta = 0$ and $\gamma = 0$, $\Delta = 1$, and the function of the control parameter Δ will disappear.

Analytical solution of degree assortativity of the model

Now we will turn to deriving analytical results of both the degree distribution and the degree assortativity in the model. The expression for assortativity is Eq. (1), and as long as the joint degree distribution $P(l, h)$ is obtained, the assortativity can be derived (Newman, 2002). In the model, each individual is assigned a status ω according to the power-law distribution $p(\omega) = c\omega^{-\alpha}$, $\omega \in [a, b]$, and the average status $\bar{\omega}$ of an individual is $\bar{\omega} = \sum_{\omega=a}^{b} \omega c\omega^{-\alpha}$. At each time step, the probability of selecting a pair of individuals with the status ω and η is $\Pi(\omega,\eta) = 2\omega\eta/(N\bar{\omega})^2$, and the probability that the two individuals establish a connection is $\Lambda(\omega,\eta) = \Pi(\omega,\eta)\Delta(\omega,\eta)$. In most empirical online social networks, the average degree of an individual is much smaller than the size of the network, i.e., the network is sparse (Leskovec and Krevl, 2015). When $t \ll N^2$, the simulated network generated by the model is a sparse network. Both the probabilities of two different individuals being chosen more than once in t time steps and one individual being chosen twice at one time step are negligible. Consequently, the duplicate links and self-connected links in the simulated network can be ignored. At each time step, once an individual is chosen and establishes a connection with another individual, the individual's degree is increased by one. Therefore, after T time steps of the simulation, the probability $P(k)$ that an individual has k neigbours is

$$P(k) = \sum_{\omega=a}^{b} p(\omega)\binom{T}{k}\varphi(\omega)^k(1-\varphi(\omega))^{T-k} \quad T \ll N^2, \tag{3}$$

in which the function $\varphi(\omega)$ are as follows

$$\varphi(\omega) = \sum_{\mu=a}^{b} \frac{2\omega}{N\bar{\omega}}\frac{\mu}{N\bar{\omega}}\Delta N c\mu^{-\alpha}, \tag{4}$$

where $\varphi(\omega)$ is the probability that an individual with the status ω is chosen and establishes a social interaction with other individual at one time step, $2\omega\mu\Delta/(N\bar{\omega})^2$ is the probability that two individuals with the statuses ω and μ respectively are chosen and establish social interaction at one time step, and $Nc\mu^{-\alpha}$ is the number of the individuals with the status μ in all N individuals.

For two individuals i, j with the status ω and η respectively, if the degrees of two connected individuals i, j are respectively l and h after T time steps of evolution. It implies that the two individuals established a connection at some time step and the corresponding probability by binomial theorem is as follows:

$$P_1(\omega,\eta) = \binom{T}{1}\left(\frac{2\omega}{N\bar{\omega}}\frac{\eta}{N\bar{\omega}}\Delta\right)\left(1-\frac{2\omega}{N\bar{\omega}}\frac{\eta}{N\bar{\omega}}\Delta\right)^{T-1}, \tag{5}$$

and i connected with other $l-1$ individuals in the rest of $T-1$ time steps and the corresponding probability is:

$$P_2(\omega, l) = \binom{T-1}{l-1}\varphi(\omega)^{l-1}(1-\varphi(\omega))^{T-1-(l-1)}, \tag{6}$$

and j connected with other $h-1$ individuals in the rest of

$T - 1 - (l - 1)$ time steps and the corresponding probability is:

$$P_3(\eta, l, h) = \binom{T - 1 - (l - 1)}{h - 1} \varphi(\eta)^{h-1}(1 - \varphi(\eta))^{T-1-(l-1)-(h-1)}. \tag{7}$$

Therefore, the joint degree distribution $P(l, h)$ of the simulation network after T time steps of evolution can be derived as follows:

$$e_{lh} = P(l, h) = \sum_{\omega=a}^{b} p(\omega) \sum_{\eta=a}^{b} p(\eta) P_1 P_2 P_3\, T \ll N^2. \tag{8}$$

With the joint degree distribution, the remaining degree distribution can be derived as:

$$q_h = \sum_l P(l, h). \tag{9}$$

The variance of the remaining degree distribution q_h is:

$$\sigma_q^2 = \sum_h h^2 q_h - \left(\sum_h h q_h\right)^2. \tag{10}$$

Finally, the degree assortativity can be derived as follows:

$$r = \frac{\sum_{l,h} lh(e_{lh} - q_l q_h)}{\sigma_q^2}. \tag{11}$$

Fig. 2 shows the comparation of degree assortativity between the analysis result and the simulation result with the change of. The analysis result and the simulation result are consistent with each other, which indicates that the analytical solution of the model is reliable. In Fig. 2, when ρ increases from 0 to 1, the degree assortativity of the simulation network changes from disassortative to assortative. The reason is that with the increase of ρ from 0 to 1, the dominant social interactions change from among individuals with the different status level to among individuals with the same status level in the evolution process of the simulation network, which causes the simulation network being from disassortative to assortative.

Verification of the model

The rise-and-fall pattern of degree assortativity in the nine empirical social networks indicates that the interactions among individuals on these networks evolves from similar to distinct status level. In our model, this shift of interaction pattern is parameterized with $\rho \in [0,1]$ in Eq. (2). In order to reproduce the rise-and-fall pattern, a cosine function of time in the initial evolution stage can therefore be adopted:

$\rho = |\cos(At + B)|,\ 1 \le t \le T$ (12)

where A and B are coefficients. In the later stage of the simulation process $t > T$, ρ approaches a constant, meaning that the dominant rule of establishing social interaction reaches a dynamic equilibrium, and consequently that the degree assortativity of simulation network tend to be stable. Fig. 3 shows the comparison of assortativity evolution between the empirical results and simulation results in the nine social networks of *Wealink*, *Wikipedia*, *Renren*, *Pussokram*, *Primary School*, *High School*, *Wikipedia English*, *Wikipedia Italian* and *Wikipedia German*. The nine empirical networks we study have sizes ranging from a few hundreds to millions of nodes, and the assortativity of them also spans a large range. Therefore, the nine empirical social networks are representative for the study of the evolution dynamics of degree assortativity in general.

In each sub-figure, the size, as well as the number of links in the simulation networks are the same with the empirical networks, and the parameters a, b, β, γ, ρ and T are assigned to appropriate values based on a large number of simulation tests on the empirical data. Since α is directly linked to the exponent of the degree distribution of the generated network, it is determined approximately equal to the power exponent of degree distribution of the simulated empirical network. In sub-figure (a) for *Wealink*, the parameters of the model are N= 11262, a= 1, b= 100, α= 2, T= 4×10^4. When $1 \le t \le T$, $\beta = 5$, $\gamma = 10$, $\rho = |\cos(-\pi/6 + 5t\pi/6T)|$, the assortativity rises in the initial evolution stage of the simulation network creation with about 2×10^3 links. This indicates that the social interactions established among the individuals with the same status level are dominant, which makes the assortativity rise. With the number of links in the simulation network being from 2×10^3 to 6×10^3, the assortativity begin to drop down. This indicates that the social interactions established among the

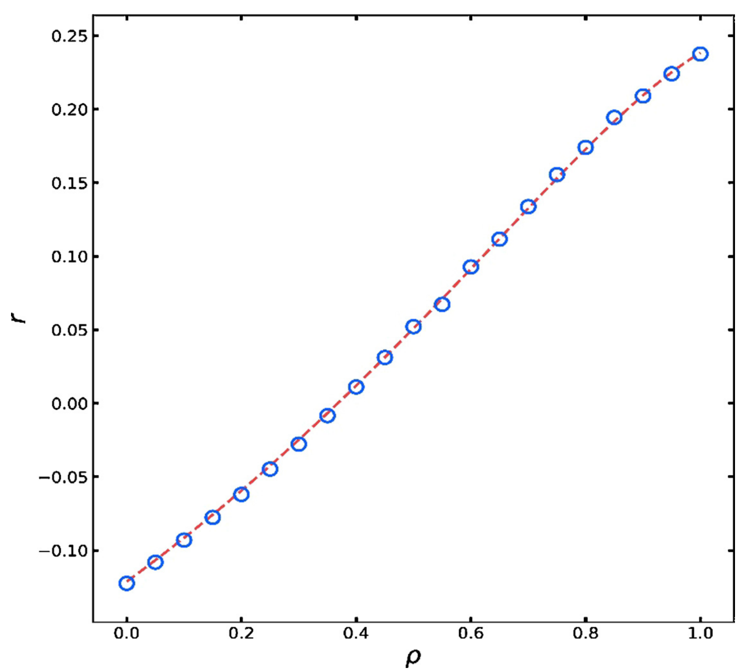

Fig. 2. The comparation of degree assortativity between the analytic result and the simulation with the change of ρ. Simulation result and analysis result are compared with N= 1×10^4, a= 1, b= 10, α= 2, T= 1×10^4, $\beta = 1$, $\gamma = 1$, $\rho \in [0,1]$. The blue hollow circle represents the simulation result and the red dash line represents the analytic result.

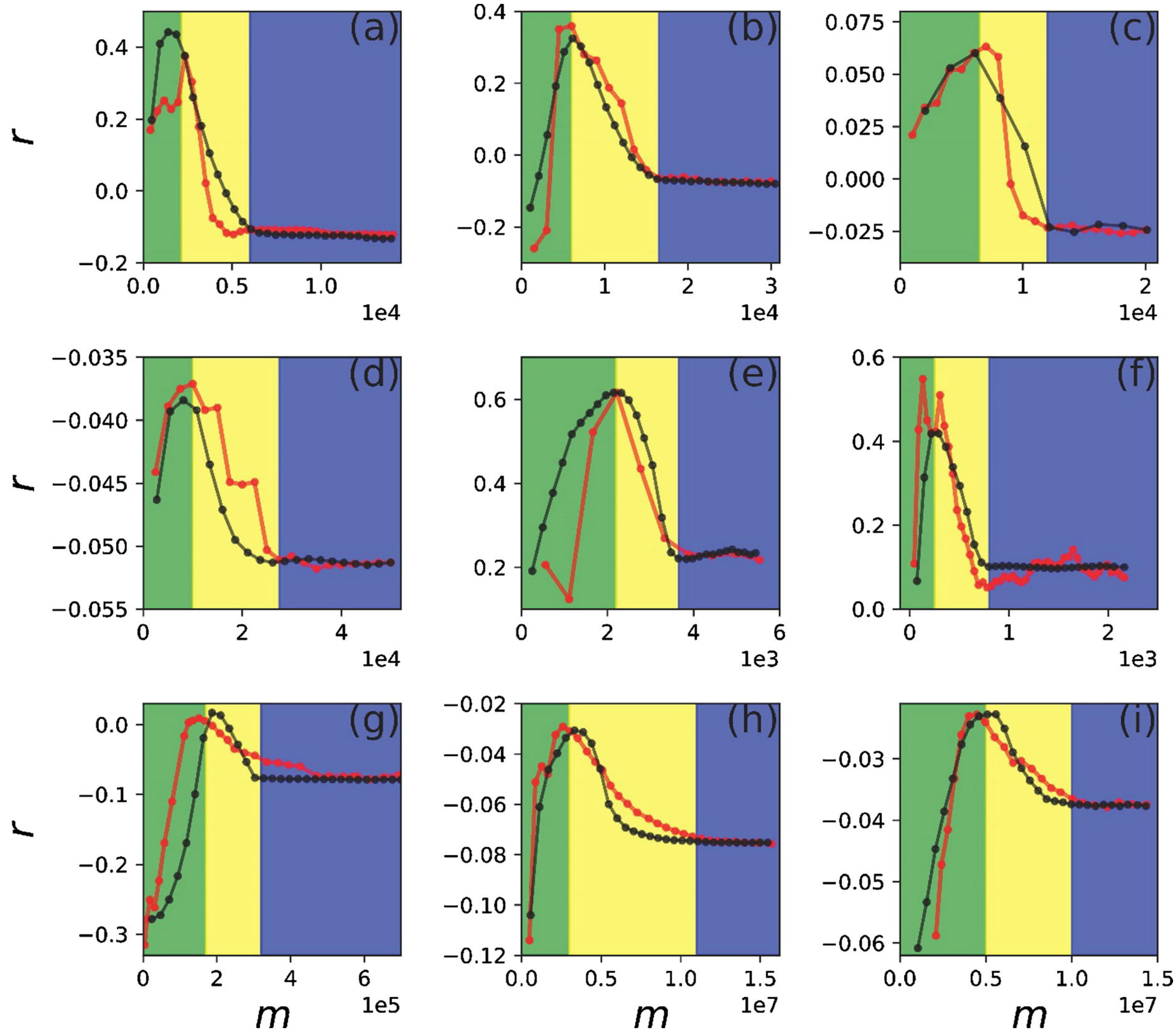

Fig. 3. The comparison of assortativity evolution between the empirical results and simulation results in the nine social networks. In each sub-figure, the size, as well as the number of links in the simulation networks are the same with the empirical networks. The red curves represent the empirical results and the black curves represent the simulation results.

individuals with different status level are dominant, which makes the assortativity drop down. When T< t≤ 1.26T, $\rho = 0.1$, $\beta = 1$, $\gamma = 1$, with the increase of links in the simulation network over 6×10^3, the assortativity of the simulation network reaches a stable value and long-lasts. This indicates that the social interactions among the individuals with the same status level and among the individuals with the different status level reach the dynamic equilibrium, which makes the assortativity of the simulation network be stable. The simulation results of the model well reproduce the evolution tendency of the assortativity in *Wealink* social network.

In others eight sub-figures, the parameters of the model are as follows. In sub-figure (b) for *Wikipedia*, N= 16435, a= 1, b= 250, α= 2, T= 8×10^4. When 1 ≤ t≤ T, $\beta = 2$, $\gamma = 10$, $\rho = |\cos(-\pi/3 + t\pi/T)|$, when T< t≤ 1.3T, $\rho = 0.3$, $\beta = 1$, $\gamma = 1$. In sub-figure (c) for *Renren*, N= 5454, a= 1, b= 100, α= 2, T= 1.5×10^4. When 1 ≤ t≤ T, $\beta = 0.01$, $\gamma = 1.5$, $\rho = |\cos(-\pi/4 + 5t\pi/6T)|$, when T< t≤ 2.1T, $\rho = 0.5$, $\beta = 1$, $\gamma = 1$. In sub-figure (d) for *Pussokram*, N= 14547, a= 1, b= 1000, α= 2, T= 3×10^4. When 1 ≤ t≤ T, $\beta = 0.003$, $\gamma = 1$, $\rho = |\cos(-5\pi/12 + 3t\pi/4T)|$, when T< t≤ 4T, $\rho = 0.6$, $\beta = 0.1$, $\gamma = 120$. In sub-figure (e) for *Primary School*, N= 238, a= 1, b= 5, α= 1, T= 1.5×10^4. When 1 ≤ t≤ T, $\beta = 2$, $\gamma = 12$, $\rho = |\cos(-\pi/4 + 5t\pi/6T)|$, when T< t≤ 1.6T, $\rho = 0.7$, $\beta = 1$, $\gamma = 120$. In sub-figure (f) for *High School*, N= 295, a= 1, b= 3, α= 1, T= 2×10^3. When 1 ≤ t≤ T, $\beta = 2$, $\gamma = 10$, $\rho = |\cos(-\pi/6 + 5t\pi/6T)|$, when T< t≤ 2.5T, $\rho = 0.7$, $\beta = 1$, $\gamma = 120$. In sub-figure (g) for *Wikipedia English*, N= 100,312, a= 1, b= 250, α= 2, T= 2.2×10^6. When 1 ≤ t≤ T, $\beta = 6$, $\gamma = 10$, $\rho = |\cos(-5\pi/12 + 10t\pi/12T)|$, when T< t≤ 1.3T, $\rho = 0.3$, $\beta = 1$, $\gamma = 1$. In sub-figure (h) for *Wikipedia Italian*, N= 1,204,009, a= 1, b= 3200, α= 2, T= 7.5×10^6. When 1 ≤ t≤ T, $\beta = 0.003$, $\gamma = 1$, $\rho = |\cos(-6\pi/12 + 14t\pi/12T)|$, when T< t≤ 5T, $\rho = 0.72$, $\beta = 1$, $\gamma = 1$. In sub-figure (i) for *Wikipedia German*, N= 1,655,809, a= 1, b= 2800, α= 2, T= 9×10^6. When 1 ≤ t≤ T, $\beta = 0.003$, $\gamma = 1.15$, $\rho = |\cos(-5\pi/12 + 9t\pi/12T)|$, when T< t≤ 3.2T, $\rho = 0.68$, $\beta = 1$, $\gamma = 1$. Although the nine social networks have very different size and assortativity levels, we can see from the nine sub-figures (a) - (i) that our model is capable of reproducing the pattern of time evolution of assortativity in all nine networks. Therefore, the model including individual status information and bidirectional preferential attachment with a control parameter can help us to understand the evolutionary mechanism of human social interaction behavior in social networks.

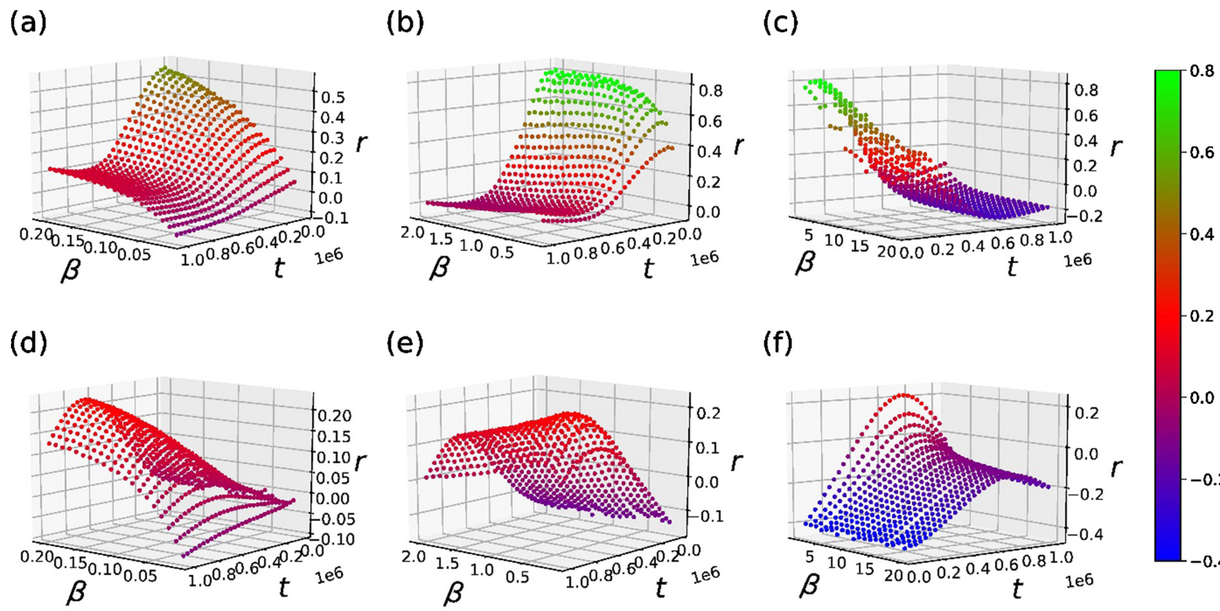

Fig. 4. The evolution of assortativity with the change of β in different simulation condition. In all sub-figures, N= 1×10^4, a= 1, b= 50, T= 1×10^6 and α= 2. (a) $\beta \in [0.01,\ 0.2]$, $\gamma = 0.1$, $\rho = |\cos(t\pi/T)|$. (b) $\beta \in [0.1,\ 2]$, $\gamma = 1$, $\rho = |\cos(t\pi/T)|$. (c) $\beta = \in[1,\ 20]$, $\gamma = 10$, $\rho = |\cos(t\pi/T)|$. The parameters in sub-figures (d), (e) and (f) are the same ones as in sub-figures (a), (b) and (c) respectively, except $\rho = |\sin(t\pi/T)|$.

Simulation of the model

In order to understand the model comprehensively, the simulations of the model are done in different simulation conditions. Fig. 4 shows the evolution of assortativity with the change of β in different simulation conditions. In Fig. 4(a), the values of β and γ are small and $\beta \in [0.01,\ 0.2]$, $\gamma = 0.1$, when $\beta = 0.01$, $\gamma = 0.1$, the assortativity fluctuates in a very small range. With an increasing, assortativity fluctuates more. This indicates that the assortativity of the simulation network is sensitive to β and γ. In Fig. 4(b) and Fig. 4(c), the value of β and γ is bigger than the ones in Fig. 4(a), the evolution of assortativity varies in a much larger range. As β and γ control the intensity of the assortativity and disassortativity of the simulation network. The bigger β or γ is, the stronger the assortativity or disassortativity is. In Fig. 4(a), 4(b) and 4(c), the network is initially assortative. On the other hand, in Fig. 4(d), 4(e) and 4(f), the network is initially disassortative. The reason is that $\rho = |\cos(t\pi/T)|$ in Fig. 4(a), 4(b) and 4(c), and $\rho = |\sin(t\pi/T)|$ in Fig. 4(d), 4(e) and 4(f). When $\rho = |\cos(t\pi/T)|$, $\rho \geq 0.5$ for t$\leq T/3$, so the social interactions established among the individuals with the same status level are dominant, which makes the simulation network assortative in the initial stage of the evolution. When $\rho = |\sin(t\pi/T)|$, $\rho \leq 0.5$ for t$\leq T/6$, so the social interactions established among the individuals with the different status level are dominant, which makes the simulation network disassortative in the initial stage of the evolution.

Fig. 5 shows the evolution of assortativity with the change of α and an interesting phenomenon was found. In the initial stage of evolution, for α∈ [1,2.5], when t$\leq 0.2 \times 10^6$, r is relatively large ($r \in [0.4,0.8]$); while when t$\geq 0.8 \times 10^6$, r is relatively small ($r \in [-0.4, -0.2]$). The assortativity and disassortativity of the simulation network can reach the greatest intensity in different evolution stages for α∈ [1,2.5]. This means that a lot of ties are formed not only among individuals with the same social status but also among individuals with different social status. This causes that a lot of information transmit not only among individuals with the same social status but also among individuals with different social status, which makes the social network being a full energy and vitality. Empirical research has shown that the range of the power exponent in Pareto wealth distribution is also α∈ [1,2.5] (Levy and Solomon, 1997; Newman, 2005; Klass et al., 2006; Arnold, 2015), echoing the applicability and generalizability of the model. Personal wealth has long been adopted as a measurement for the quantification of individual social status in human society (Angle, 1986). Therefore, we conjecture that the individual social status distribution in the human society may drive the social evolution in a way of self-optimization to promote the social interaction and information transmission among individuals. Highly efficient information transmission among individuals can greatly promote the progress and the development of the human society (Hamelink, 1997).

Fig. 6 shows the evolution of assortativity with the change of b. With b∈ [1, 16], we can observe a phase transition. When $a = 1$ and $b = 1$, the assortativity r is a constant and is equal zero in the evolution process of the simulation network, while when $a = 1$ and $b > 1$, the assortativity r is mutational to take place clear change in the evolution process of the simulation network. This reason is that when $a = 1$, $b = 1$, all individuals are at the same status level and there is no status gap. This causes $\lambda_1 = 1$, $\lambda_2 = 1$, $\Delta = 1$, and the function of the control parameter Δ disappears. The social interactions among individuals are stochastic and the significance of bidirectional preferential attachment also disappears. Consequently, the simulation network is neither assortative nor disassortative in the evolutionary process, i.e., the assortativity r is always zero. When $a = 1$ and $b > 1$, the gap in status appears and the control parameter Δ begins to take effect, which controls the simulation network to be assortative or disassortative in the evolution process. It is worth noting that when $a = 1$ and $b > 1$, with $\beta = 0$ and $\gamma = 0$, $\Delta = 1$, and the function of the control parameter Δ will also disappear, but the bidirectional preferential attachment is still present. This will cause that the social interactions prefer to be established among individuals with the different social status in the evolution process, which makes the simulation network disassortative. These results show that our model is accurate and effective in controlling the assortativity and disassortativity of the simulated networks and suggest that a skewed distribution of social status can play an important role in the emergence of degree correlations of social networks.

Conclusion and discussion

Using nine temporal empirical social network datasets from *Wealink*, *Wikipedia*, *Renren*, *Pussokram*, *Primary School*, *High School*,

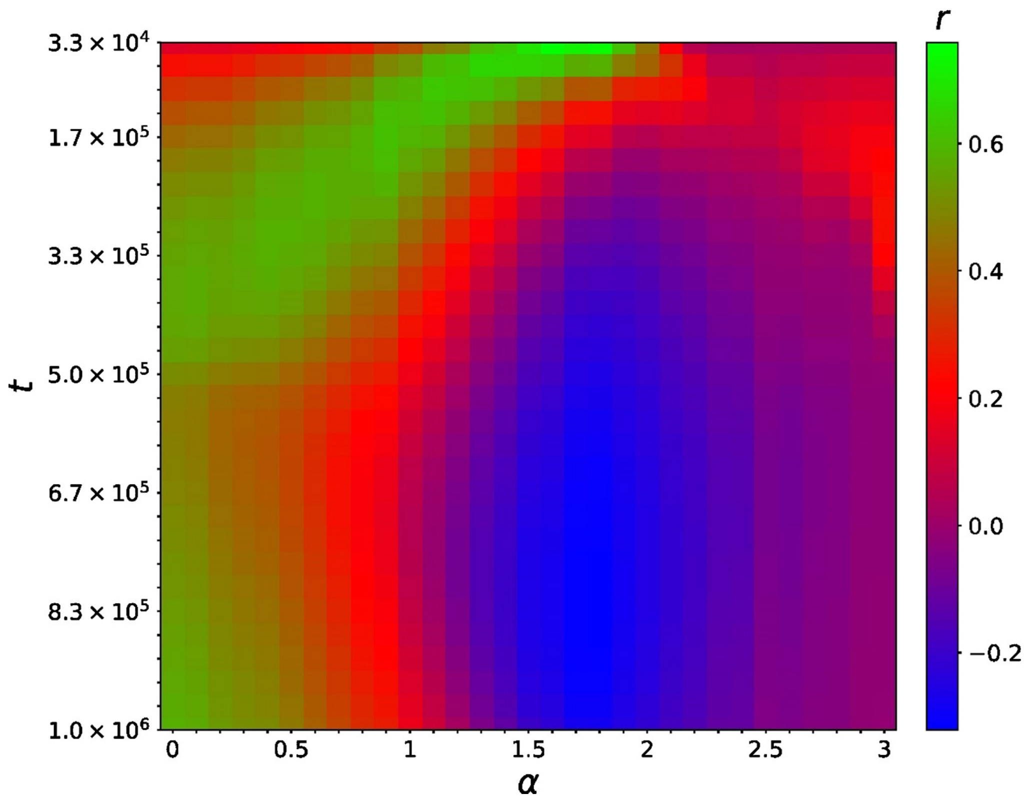

Fig. 5. The evolution of assortativity with the change of α in different simulation conditions. Simulations are done with N= 1×10^4, a= 1, b= 50, α∈ [0,3], T= 1×10^6, $\beta = 15$, $\gamma = 10$, ρ. $= |\cos(t\pi/T)|$

Wikipedia English, *Wikipedia Italian* and *Wikipedia German*, we have uncovered a surprisingly consistent pattern on the evolution of network assortativity, that the assortativity of social networks increases rapidly at the beginning of network growth, then decays to an equilibrium after certain links being created. Based on Pareto wealth distribution and bidirectional preferential attachment mechanism, we propose a temporal network model with a control parameter and provide the evolutionary analytical solutions of both the degree distribution and the assortativity in the model. The simulation results indicate that the model can reproduce the universal evolution pattern of assortativity of the social networks. In order to understand the model comprehensively, we scanned its parameter space. These simulation results indicate that

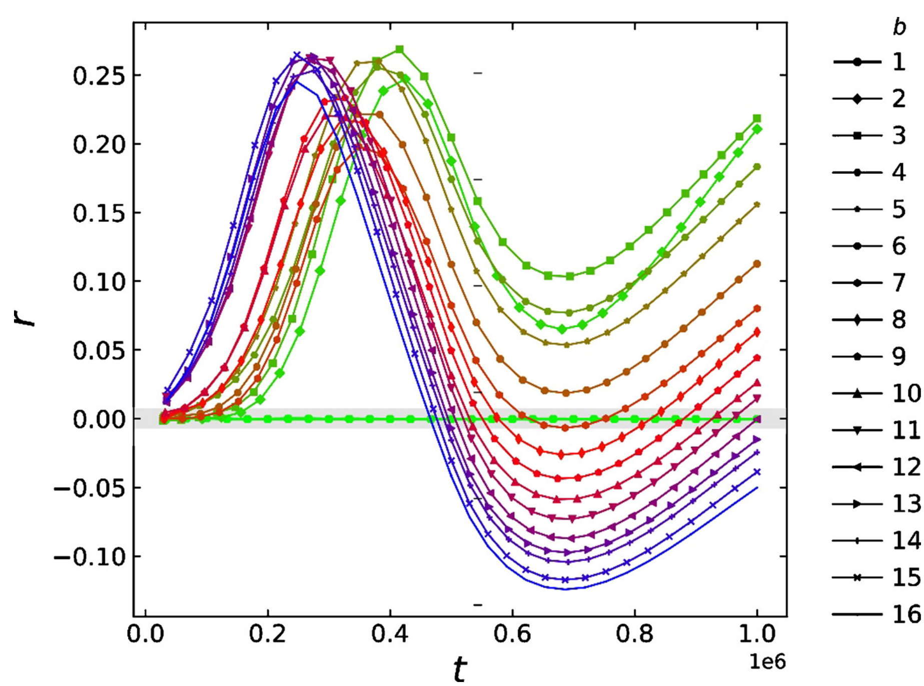

Fig. 6. The evolution of assortativity with the change of b in different simulation conditions. Simulations are done with N= 1×10^4, a= 1, b∈ [1, 16], α= 2, T= 1×10^6, $\beta = 15$, $\gamma = 10$, ρ. $= |\cos(t\pi/T)|$

Pareto distributions in the human society may drive the social evolution in a way of self-optimization to promote the social interaction among individuals and the wealth gap plays an important role in both assortative and disassortative regimes of social networks. Therefore, a Pareto distribution of social status and a bidirectional preferential attachment mechanism help us to understand the evolution of social networks.

As far as we know, our model is the first that can mimic the universal evolution pattern of the assortativity in the social network. Its simplicity allows more specific mechanisms to be integrated into future modeling efforts. Above all, the bidirectional preferential attachment model with the individual status information and a control parameter implies us the possible and worthwhile efforts in exploring the unified mechanisms behind various networks. Because of our successful modeling of the universal evolution pattern of assortativity we think this could be a more general model of social network formation. This work can be the basis for future exploration about the relation between social interaction and information diffusion.

Acknowledgements

XL is supported by the National Natural Science Foundation of China (82041020, 71771213, 91846301), the Sichuan Science and Technology Plan Project (2020YFS0007), and the Hunan Science and Technology Plan Project (2017RS3040, 2018JJ1034, 2019GK2131). BZ is funded by the Natural Science Foundation of China (61503159) and Jiangsu University Overseas Training Program. PH is supported by JSPS KAKENHI (JP 18H01655) and the Grant for Basic Science Research Projects by the Sumitomo Foundation.

References

Abbate, A., De Benedictis, L., Fagiolo, G., Tajoli, L., 2017. Distance-varying assortativity and clustering of the international trade network. Networks Science forthcoming.
Allen-Perkins, A., Pastor, J.M., Estrada, E., 2017. Two-walks degree assortativity in graphs and networks. Appl. Math. Comput. 311, 262–271.
Angle, J., 1986. The surplus theory of social stratification and the size distribution of personal wealth. Soc. Forces 65 (2), 293–326.
Arnold, B.C., 2015. Pareto Distribution. John Wiley and Sons, Ltd.
Badham, J., Stocker, R., 2010. The impact of network clustering and assortativity on epidemic behaviour. Theor. Popul. Biol. 77 (1), 71–75.
Barabási, A.L., Albert, R., 1999. Emergence of scaling in random networks. Science 286 (5439), 509–512.
Barrat, A., Barthelemy, M., Vespignani, A., 2008. Dynamical Processes on Complex Networks. Cambridge University Press.
Becker, J., Brackbill, D., Centola, D., 2017. Network dynamics of social influence in the wisdom of crowds. Proc. Natl. Acad. Sci. 114 (26), 5070–5076.
Boguná, M., Pastor-Satorras, R., Vespignani, A., 2003. Absence of epidemic threshold in scale-free networks with degree correlations. Phys. Rev. Lett. 90 (2), 028701.
Bordieu, P., 1984. Distinction: A Social Critique of the Judgement of Taste. Routledge.
Catanzaro, M., Caldarelli, G., Pietronero, L., 2004. Assortative model for social networks. Phys. Rev. E 70 (3), 037101.
Ciglan, M., Laclavík, M., Nørvåg, K., 2013. On Community Detection in Real-world Networks and the Importance of Degree Assortativity. In Proceedings of the 19th ACM SIGKDD International Conference on Knowledge Discovery and Data Mining 1007-1015. ACM.
Colizza, V., Flammini, A., Serrano, M.A., Vespignani, A., 2006. Detecting rich-club ordering in complex networks. Nat. Phys. 2 (2), 110.
Del Vicario, M., Zollo, F., Caldarelli, G., Scala, A., Quattrociocchi, W., 2017. Mapping social dynamics on Facebook: the brexit debate. Soc. Networks 50, 6–16.
Di Bernardo, M., Garofalo, F., Sorrentino, F., 2007. Effects of degree correlation on the synchronization of networks of oscillators. Int. J. Bifurc. Chaos 17 (10), 3499–3506.
Eguiluz, V.M., Klemm, K., 2002. Epidemic threshold in structured scale-free networks. Phys. Rev. Lett. 89 (10), 108701.
Estrada, E., 2011. Combinatorial study of degree assortativity in networks. Phys. Rev. E 84 (4), 047101.
Faraj, S., Jarvenpaa, S.L., Majchrzak, A., 2011. Knowledge collaboration in online communities. Organ. Sci. 22 (5), 1224–1239.
Hamelink, C.J., 1997. New Information and Communication Technologies, Social Development and Cultural Change Vol. 86 United Nations Research Institute for Social Development, Geneva.
Holme, P., Edling, C.R., Liljeros, F., 2004. Structure and time evolution of an Internet dating community. Soc. Networks 26 (2), 155–174.
Holme, P., Zhao, J., 2007. Exploring the assortativity-clustering space of a network's degree sequence. Phys. Rev. E 75 (4), 046111.
Hu, H.B., Wang, X.F., 2009. Evolution of a large online social network. Phys. Lett. A 373 (12), 1105–1110.
Jiang, J., Qu, Y., Yu, S., Zhou, W., Wu, W., 2016. Studying the Global Spreading Influence and Local Connections of Users in Online Social Networks. In Computer and Information Technology (CIT), IEEE International Conference on. pp. 431–435.
Kim, J., Hastak, M., 2018. Social network analysis. International Journal of Information Management: The Journal for Information Professionals 38 (1), 86–96.
Klass, O.S., Biham, O., Levy, M., Malcai, O., Solomon, S., 2006. The Forbes 400 and the Pareto wealth distribution. Econ. Lett. 90 (2), 290–295.
Lee, E., Holme, P., 2017. Social contagion with degree-dependent thresholds. Phys. Rev. E 96, 012315.
Leskovec, J., Krevl, A., 2015. Large Network Dataset Collection. snap.stanford.edu.
Levy, M., Solomon, S., 1997. New evidence for the power-law distribution of wealth. Phys. A Stat. Mech. Its Appl. 242 (1), 90–94.
Li, M., Guan, S., Wu, C., Gong, X., Li, K., Wu, J., Lai, C.H., 2014. From sparse to dense and from assortative to disassortative in online social networks. Sci. Rep. 4.
Litvak, N., van der Hofstad, R., 2013. Uncovering disassortativity in large scale-free networks. Phys. Rev. E 87, 022801.
Mastrandrea, R., Fournet, J., Barrat, A., 2015. Contact patterns in a high school: a comparison between data collected using wearable sensors, contact diaries and friendship surveys. PLoS One 10 (9), e0136497.
McAuley, J.J., da Fontoura Costa, L., Caetano, T.S., 2007. Rich-club phenomenon across complex network hierarchies. Appl. Phys. Lett. 91 (8), 084103.
Murphy, C., Allard, A., Laurence, E., St-Onge, G., Dubé, L.J., 2018. Geometric evolution of complex networks with degree correlations. Phys. Rev. E 97 (3), 032309.
Newman, M.E.J., 2002. Assortative mixing in networks. Phys. Rev. Lett. 89 (20), 208701.
Newman, M.E.J., 2005. Power laws, Pareto distribution and Zipf's law. Contemp. Phys. 46 (5), 323–351.
Newman, M.E.J., Park, J., 2003. Why social networks are different from other types of networks. Phys. Rev. E 68 (3), 036122.
Noldus, R., van Mieghem, P., 2015. Assortativity in complex networks. J. Complex Netw. 3 (4), 507–542.
Ohtsuki, H., Hauert, C., Lieberman, E., Nowak, M.A., 2006. A simple rule for the evolution of cooperation on graphs and social networks. Nature 441 (7092), 502.
Opsahl, T., Colizza, V., Panzarasa, P., Ramasco, J.J., 2008. Prominence and control: the weighted rich-club effect. Phys. Rev. Lett. 101 (16), 168702.
Opsahl, T., Agneessens, F., Skvoretz, J., 2010. Node centrality in weighted networks: generalizing degree and shortest paths. Soc. Networks 32 (3), 245–251.
Pareto, V., 1964. Cours d'économie politique. Librairie Droz.
Stehlé, J., Voirin, N., Barrat, A., Cattuto, C., Isella, L., Pinton, J.F., Quaggiotto, M., Broeck, W., Régis, C., Lina, B., Vanhems, P., 2011. High-resolution measurements of face-to-face contact patterns in a primary school. PLoS One 6 (8), e23176.
Strogatz, S.H., 2001. Exploring complex networks. Nature 410 (6825), 268.
Toivonen, R., Onnela, J.P., Saramäki, J., Hyvönen, J., Kaski, K., 2006. A model for social networks. Phys. A Stat. Mech. Its Appl. 371 (2), 851–860.
Vaquero, L.M., Cebrian, M., 2013. The rich club phenomenon in the classroom. Sci. Rep. 3, 1174.
Xulvi-Brunet, T., Sokolov, I., 2005. Changing correlations in networks: assortativity and disassortativity. Acta Phys. Pol. B 36 (5), 1431–1455.
Zhou, D., Stanley, H.E., D'Agostino, G., Scala, A., 2012. Assortativity decreases the robustness of interdependent networks. Phys. Rev. E 86 (6), 066103.
Zhou, B., Yan, X.Y., Xu, X.K., Xu, X.T., Wang, N., 2018. Evolutionary of online social networks driven by pareto wealth distribution and bidirectional preferential attachment. Phys. A Stat. Mech. Its Appl. 507, 427–434.